ARCADIA aeroacoustic design code

Sergio Campobasso
Mihai Duta
Mike Giles
Lorenzo Lafronza
Alistair Laird

May 12th, 2003

Overview

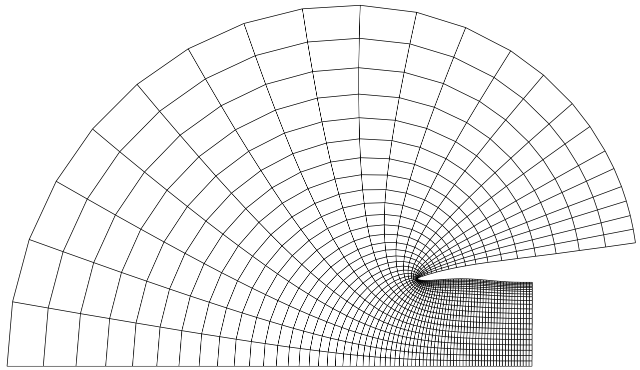
Oxford role is to contribute expertise in CFD and the use of adjoints for design sensitivities through two codes:

- HYDRA – existing Euler/Navier-Stokes code for a range of applications
- ARCADIA – new potential flow code for fan tone noise

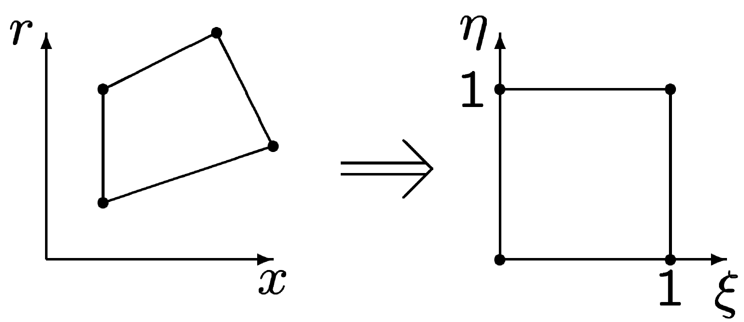
This presentation will focus on ARCADIA, and its novel use of asymptotics and adjoint methods.

Axisymmetric Geometry

2D axisymmetric grid



with each cell mapped to a unit square



Axisymmetric Geometry

Coordinates and potential flow solution are interpolated from nodal values:

$$
\begin{aligned}
x(\xi, \eta) &= \sum_n x_n \, N_n(\xi, \eta), \\
r(\xi, \eta) &= \sum_n r_n \, N_n(\xi, \eta), \\
\phi(\xi, \eta) &= \sum_n \phi_n \, N_n(\xi, \eta), \\
\widehat{\phi}(\xi, \eta) &= \sum_n \widehat{\phi}_n \, N_n(\xi, \eta),
\end{aligned}
$$

Putting this into the weak form of the potential flow equations gives discrete systems of equations for the nodal values.

Axisymmetric Geometry

Steady flow analysis leads to a coupled system of nonlinear equations

$$R(\phi) = 0$$

which is solved by Newton iteration

$$K_\nu \Delta\phi_\nu = -R_\nu$$

where $K_\nu = \left(\frac{\partial R}{\partial \phi}\right)_\nu$.

Axisymmetric Geometry

The unsteady flow analysis for a prescribed circumferential mode number

$$\exp(\mathrm{i}\omega t + \mathrm{i}\kappa\theta)\, \widehat{\phi}(x, r)$$

leads to a linear system of equations

$$(-\omega^2 M + \mathrm{i}\omega C + K + \kappa^2 K^\theta)\, \widehat{\phi} = \widehat{f}$$

with the forcing term $\widehat{f}$ coming from the modal boundary condition on the fan face.

Currently, ARCADIA does not have an acoustic liner model, and the far-field non-reflecting boundary conditions are low-order.

Non-axisymmetric Geometry

Three ways of handling non-axisymmetry:

- 3D finite elements, with hexahedral elements mapped to unit cubes

$$\phi(\xi, \eta, \zeta) = \sum_n \phi_n \, N_n(\xi, \eta, \zeta)$$

- spectral elements with $x, r, \phi, \widehat{\phi}$ expressed as Fourier series

$$\phi(\xi, \eta, \theta) = \sum_{m,n} \phi_{m,n} \, exp(\mathrm{i} m\theta) \, N_n(\xi, \eta)$$

- asymptotic analysis, like the spectral approach but assuming small asymmetry

Non-axisymmetric Geometry

Two kinds of error:

- 3D $-$ spectral $=$

  error due to circumferential resolution
- asymptotic $-$ spectral $=$

  error due to linearisation

Pros and cons:

- 3D analysis straightforward in principle, but needs a fine grid so costly in practice
- spectral elements require many fewer unknowns, but tricky to solve the equations
- asymptotic analysis uses 2D calculations so cheap, but can it handle large perturbations?

Non-axisymmetric Geometry

Steps in asymptotic analysis:

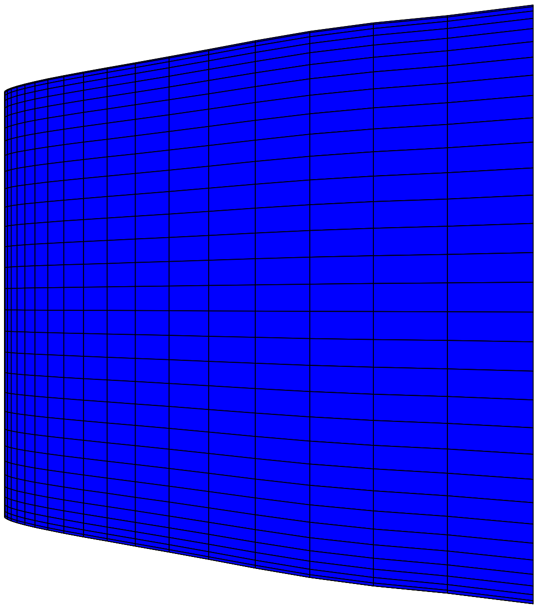
- start with decomposition of geometry into axisymmetric average plus perturbation
- compute ϕ_0, axisymmetric average flow field
- solve a linear perturbation equation for each circumferential flow field perturbation mode

$$L_m\,\phi_m + A_m\,x_m + B_m\,r_m = 0,$$

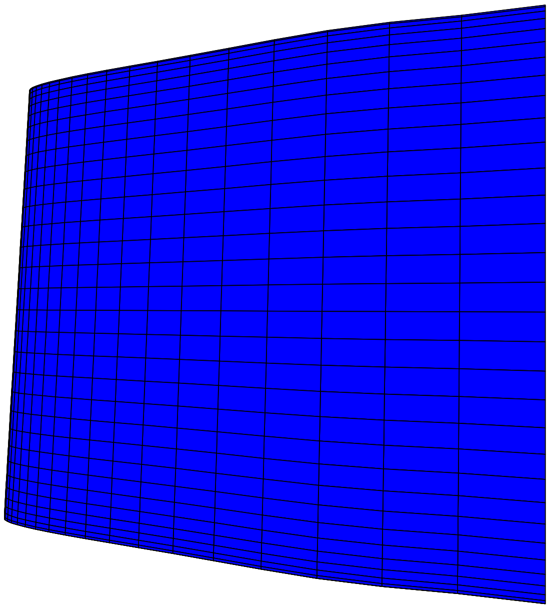
- compute $\widehat{\phi}_0$, axisymmetric part of linear unsteady solution
- solve separate equations for the other modes

$$\widehat{L}_m\,\widehat{\phi}_m + \widehat{A}_m\,x_m + \widehat{B}_m\,r_m + \widehat{C}_m\,\phi_m = 0$$

Steady validation

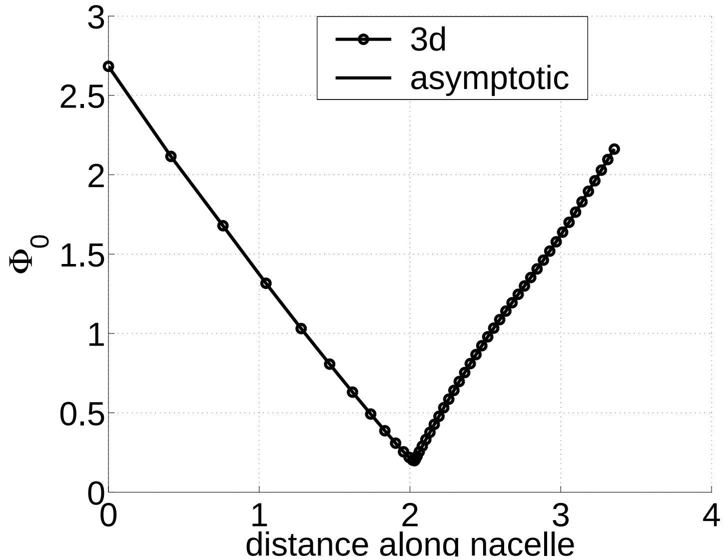


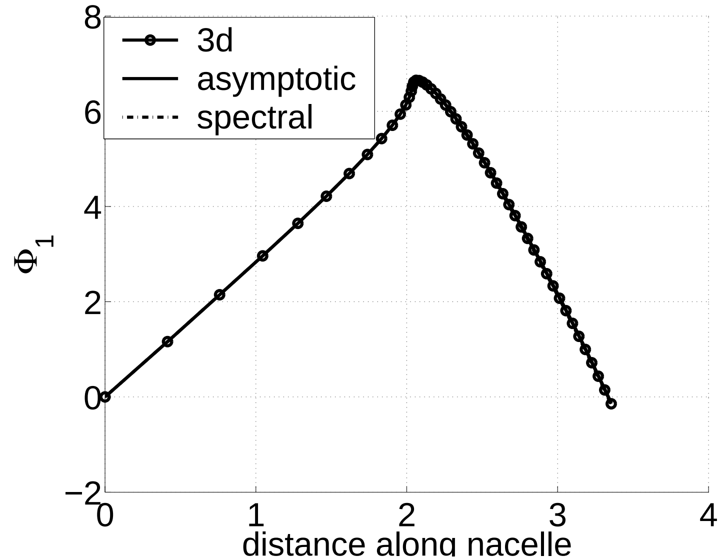
Axisymmetric



Asymmetric

Steady validation

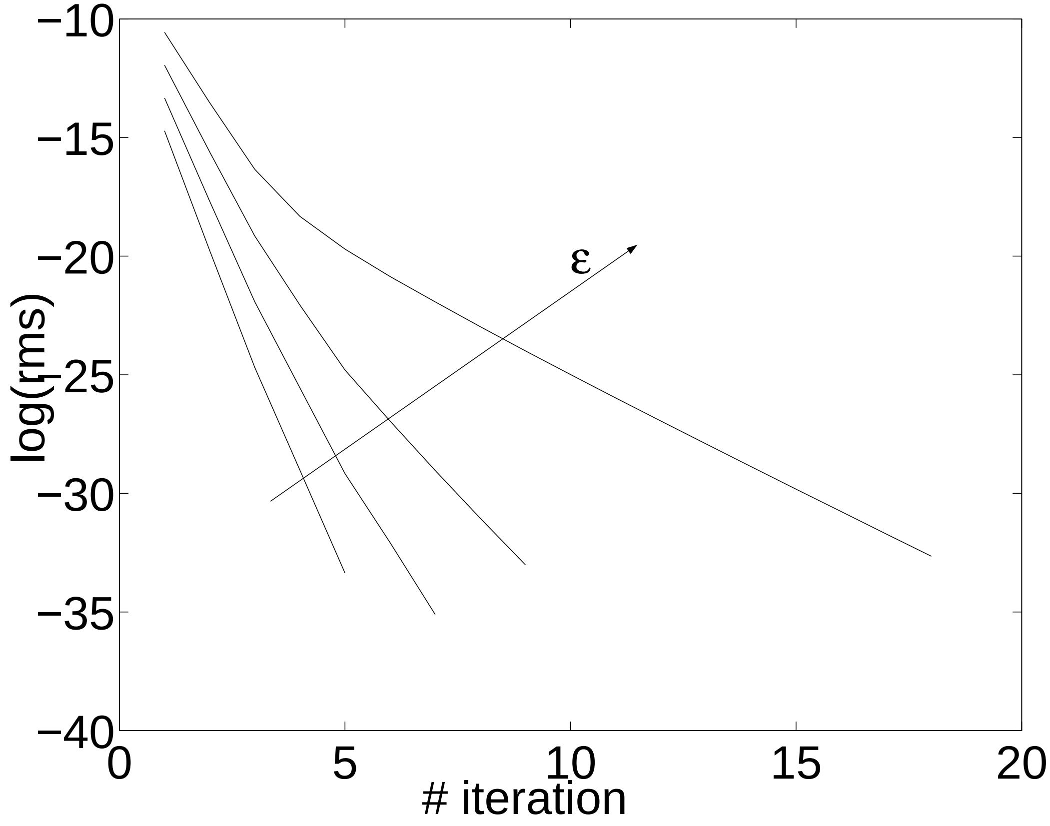




$0th$ harmonic

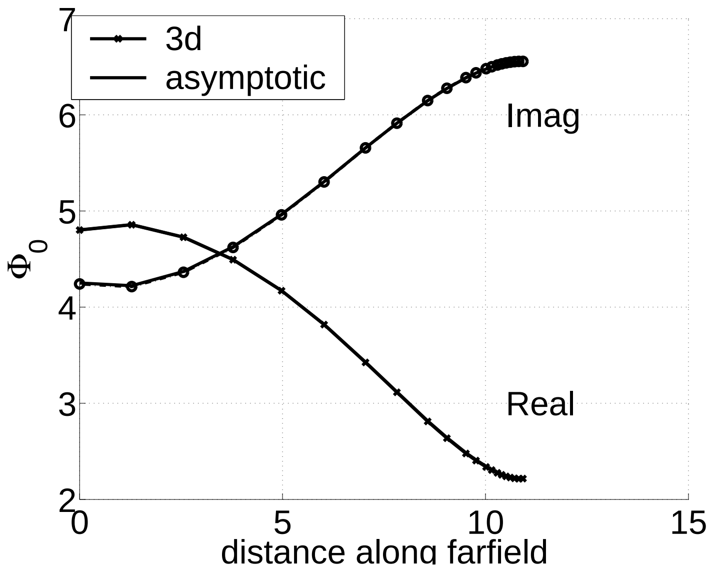
$1st$ harmonic

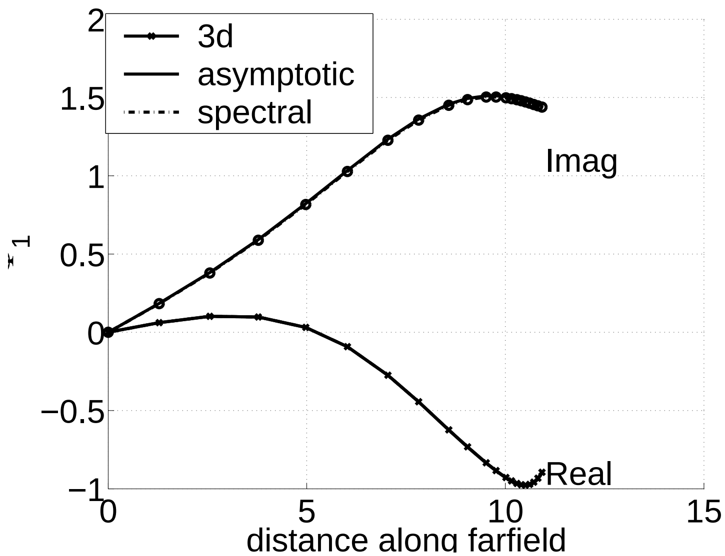
Steady validation



iterative convergence of spectral solver

Unsteady validation





$0th$ harmonic

$1st$ harmonic

Adjoint Equations

For some output functional $J(x, r, \phi)$ the sensitivity to changes in a design variable α is

$$\frac{\mathrm{d}J}{\mathrm{d}\alpha} = \sum_m \left(\frac{\partial J}{\partial x_m} \frac{\partial x_m}{\partial \alpha} + \frac{\partial J}{\partial r_m} \frac{\partial r_m}{\partial \alpha} + \frac{\partial J}{\partial \phi_m} \frac{\partial \phi_m}{\partial \alpha} \right)$$

The flow sensitivity is given by

$$L_m \frac{\partial \phi_m}{\partial \alpha} + A_m \frac{\partial x_m}{\partial \alpha} + B_m \frac{\partial r_m}{\partial \alpha} = 0,$$

Adjoint Equations

Hence, by defining the adjoint variables v_m by the equation

$$L_m^H v_m = -\left(\frac{\partial J}{\partial \phi_m}\right)^H,$$

one obtains

$$\frac{\mathrm{d}J}{\mathrm{d}\alpha} = \sum_m \left\{ \left(v_m^H A_m + \frac{\partial J}{\partial x_m}\right)\frac{\partial x_m}{\partial \alpha} + \left(v_m^H B_m + \frac{\partial J}{\partial r_m}\right)\frac{\partial r_m}{\partial \alpha} \right\}.$$

This gives a very efficient way of computing the sensitivity to multiple design variables.

Adjoint Equations

For functionals $\widehat{J}(x, r, \phi, \widehat{\phi})$ depending on the unsteady flow field, there is a two-stage process.

First one solves the unsteady adjoint equations

$$\widehat{L}_m^H \, \widehat{v}_m = -\left(\frac{\partial \widehat{J}}{\partial \widehat{\phi}_m}\right)^H$$

and then one solves the steady adjoint equations

$$L_m^H \, v_m = -\left(\widehat{v}_m^H \, \widehat{C}_m + \frac{\partial \widehat{J}}{\partial \phi_m}\right)^H$$

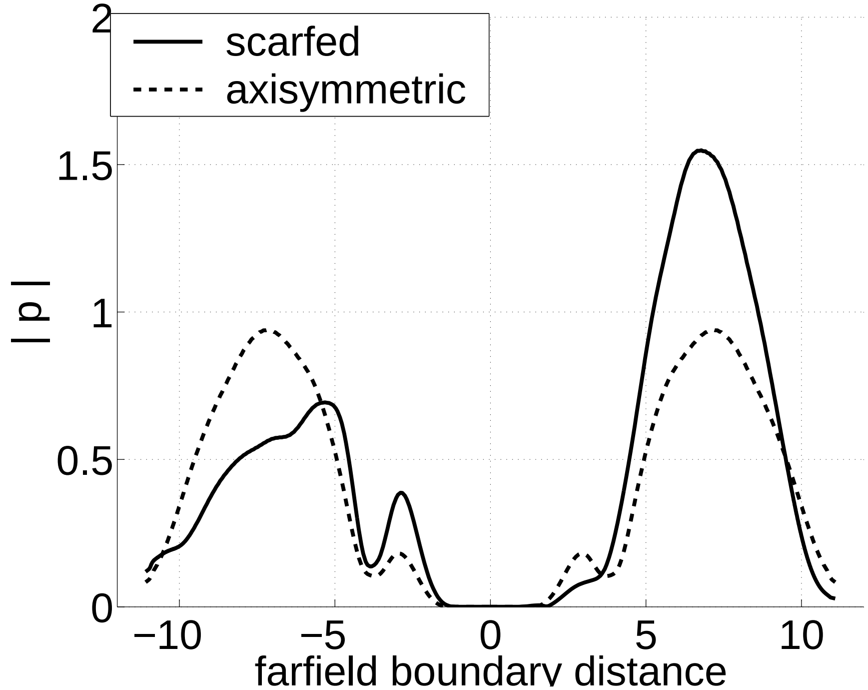
Adjoint Equations

The final sensitivity result is then

$$\frac{\mathrm{d}\widehat{J}}{\mathrm{d}\alpha} = \sum_m \left\{ \left(v_m^H A_m + \widehat{v}_m^H \widehat{A}_m + \frac{\partial \widehat{J}}{\partial x_m} \right) \frac{\partial x_m}{\partial \alpha} + \left(v_m^H B_m + \widehat{v}_m^H \widehat{B}_m + \frac{\partial \widehat{J}}{\partial r_m} \right) \frac{\partial r_m}{\partial \alpha} \right\}.$$

Because we explicitly assemble all of the necessary matrices for the asymptotic analysis, the implementation of the adjoint analysis is trivial. This will be used with gradient-based optimisation in GEODISE.

Objective



Reduce the noise radiated towards the ground by re-designing the nacelle (negative scarfing).